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FUNCTIONAL GRAMMAR OF MODAL MUSIC: TOWARDS A UNIFIED THEORY OF MODAL ORGANIZATION

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Abstract

The present study proposes a theoretical framework for representing modal music through functional organization rather than solely through acoustic or statistical properties of pitch sequences. Despite substantial historical and cultural differences, Azerbaijani mugham, Turkish makam, Indian raga, and Gregorian chant exhibit functional analogies in the organization of melodic development around stable reference points, secondary centers, characteristic trajectories, and recurrent melodic structures. Existing Music Information Retrieval (MIR) approaches primarily operate on acoustic features, pitch distributions, or statistical regularities and may therefore remain limited in capturing higher-level functional logic. To address this gap, we introduce the Functional Grammar of Modal Music (FGMM), a generalized representation in which modal melodies are modeled as trajectories through a space of functional states constrained by stylistic rules and transition structures. FGMM identifies four minimal components: a primary functional center, a secondary functional center, a developmental trajectory, and recurrent melodic patterns. The framework is formulated independently of any single musical tradition and is

intended to support structural analysis, automatic classification, cross-cultural comparison, and interpretable algorithmic generation. We further outline a computational pipeline and quantitative descriptors for future implementation. The study should be regarded as a conceptual and mathematical foundation for empirical validation rather than as a completed computational system. Its principal contribution is the identification of underexplored functional analogies between historically independent modal traditions and the proposal of a common representational language linking comparative musicology, MIR, and generative artificial intelligence.
Keywords: Modal music, Functional Grammar, Music Information Retrieval, Computational Musicology, Melodic Trajectories, Generative Artificial Intelligence.

1. Introduction

Modal musical traditions represent some of the most sophisticated forms of melodic organization developed in human culture. Unlike harmonic systems primarily governed by chord progressions and tonal functions, modal traditions rely on controlled melodic development within a framework of culturally established constraints and expectations [28].

Examples of such traditions include Azerbaijani mugham [15], Turkish makam [37], Indian raga [19], Gregorian chant [16], Byzantine chant [41], and Persian dastgah [12]. Despite substantial geographical separation and independent historical evolution, many of these systems exhibit striking similarities in their internal organization. Stable reference tones, secondary centers of attraction, characteristic developmental trajectories, and recurrent melodic structures appear repeatedly under different theoretical terminologies and pedagogical traditions [13; 29; 30; 32; 38].

Traditional musicology has extensively documented these systems within their respective cultural contexts. Mugham theory has been developed through the work of Hajibeyov and subsequent Azerbaijani musicologists [15; 18]. Turkish makam theory has developed a rich formal terminology including concepts such as durak, güçlü, and seyir (20; 37), while Indian musicology has described analogous notions through sa, vadi, samvadi, and pakad [5; 19]. Gregorian chant scholarship has similarly identified stable final tones, reciting tones, and characteristic melodic formulae governing modal development [1; 10].

At the same time, modern computational musicology and Music Information Retrieval (MIR) have made substantial progress in the extraction and analysis of musical information from audio signals [7; 10; 26; 35; 39]. Existing approaches successfully identify pitch contours, interval distributions, rhythmic structures, and timbral characteristics, enabling large-scale analysis and classification of musical corpora [3; 4; 11; 34].

However, an important representational gap remains.Most MIR systems operate either at the level of acoustic signals or at the level of statistical descriptions derived from pitch sequences. While highly effective for many applications, such approaches often remain limited in capturing the higher-order functional organization that musicians intuitively recognize during performance and improvisation [6; 14; 21].

An experienced mugham performer does not select the next note solely on the basis of local pitch probabilities. Likewise, a Gregorian cantor or a raga performer does not navigate a melody as an unconstrained sequence of pitch events. Musical development is instead governed by a higher-level system of functional constraints that defines which continuations are stylistically acceptable within a particular modal tradition [2; 27; 31].

The present study originates from the hypothesis that these higher-order constraints may possess a level of abstraction transcending individual musical cultures. Interestingly, similar situations arise in many scientific disciplines where apparently unrelated systems eventually reveal common underlying organizational principles.

Different musical traditions appear to optimize different aspects of human perception and social interaction. Rhythmic traditions such as marches and work songs facilitate synchronization and collective coordination, while dance music emphasizes motor entrainment and non-verbal interaction between participants. Harmonic traditions frequently organize emotional expectation through tension and resolution mechanisms.

Certain musical traditions, however, appear to pursue a different objective: the maintenance of prolonged attention through the internal development of melody itself. This category includes modal and improvisational traditions such as mugham, raga, Gregorian chant, and possibly certain forms of jazz.

Despite substantial differences in cultural origin and theoretical vocabulary, these traditions demonstrate a remarkable ability to sustain attentive listening over extended periods while relying relatively little on harmonic progression or repetitive rhythmic structures.

This observation motivated the hypothesis that musical systems solving a similar perceptual problem may converge toward similar organizational principles despite historical independence. The authors do not claim that such convergence necessarily implies common historical origin. Instead, the working assumption of the present study is that similar functional requirements may sometimes produce similar organizational solutions.

Rather than focusing primarily on differences between modal systems, we ask a complementary question:

Which organizational principles emerge repeatedly when different civilizations attempt to construct long-duration, attention-sustaining, and spiritually oriented melodic systems?

This perspective shifts the discussion from historical genealogy to functional convergence.

The central hypothesis of this paper is that several historically independent modal traditions exhibit convergent solutions to similar organizational challenges and that these solutions may be described using a common functional language.

To formalize this idea, we introduce the concept of *Functional Grammar of Modal Music (FGMM)*. Instead of representing melodies exclusively through pitch sequences or statistical distributions, FGMM models modal music as movement through a space of functional states governed by constraints and transition rules.

The proposed framework identifies four minimal functional components that appear, under different names and terminologies, in multiple modal traditions:

- a primary functional center;
- a secondary functional center;
- a characteristic developmental trajectory;
- a system of recurrent melodic patterns.

These components form the basis of a generalized representation capable of describing modal organization independently of any specific cultural tradition.
The choice of four functional components should not be interpreted as uniquely correct or theoretically complete. Nor should these components be understood as arbitrary theoretical constructs introduced solely for the purposes of the present study.

Rather, the proposed representation emerged from a comparative analysis of concepts already recognized by musicologists and practitioners within individual modal traditions. Terms such as durak, güçlü, seyir, and ajnas in Turkish makam theory [20; 37], finalis, tenor, and cadential formulae in Gregorian chant [1; 10], and sa, vadi, and pakad in Indian raga theory [5; 19; 30] have long occupied central positions within their respective theoretical systems.

The contribution of the present work lies not in introducing these functional categories themselves but in identifying their structural analogies across historically independent musical cultures and organizing them within a unified representational framework.

Unlike previous studies that primarily emphasize differences between modal systems, the present approach focuses on identifying previously underexplored functional analogies between independently evolved musical traditions.

The proposed framework is intended to support several classes of applications, including structural analysis of modal melodies, automatic classification of modal traditions, comparative

musicology, music information retrieval, and interpretable algorithmic generation of stylistically valid modal material.

Since the present work aims to identify common organizational principles rather than to analyze a particular musical corpus, the adopted methodology is comparative and conceptual rather than experimental. The following section therefore describes the logic of the comparative procedure underlying the proposed framework.

The remainder of this paper is organized as follows. Section 2 describes the comparative methodology employed in the present work. Section 3 discusses the role of structural constraints in musical systems. Section 4 examines functional analogies between major modal traditions. Section 5 introduces the Functional Grammar of Modal Music framework and its mathematical representation. Section 6 presents a computational pipeline for extracting functional representations from audio recordings. Section 7 discusses quantitative descriptors derived from FGMM and outlines possible directions for future computational implementation and empirical validation.

2. Methodology

The present work is conceptual and comparative in nature and does not rely on experimental measurements or statistical analysis of musical corpora. Instead, it adopts a comparative methodology in which recurring structural similarities across independently developed systems are examined as potential indicators of underlying organizational principles. Comparable forms of structural and convergent reasoning have been employed in comparative social research, structural anthropology, and studies of convergent evolution [9; 24; 25; 33]. Conceptual studies of this type occupy an unusual but important position in scientific development: historically, theoretical frameworks have often preceded the availability of methods capable of testing them experimentally. The methodological procedure adopted in this study consists of five sequential stages.

First, several modal traditions characterized by substantial geographical separation and largely independent historical development were selected for analysis. The primary systems considered in the present work include Azerbaijani mugham [18], Turkish makam [20; 37], Indian raga [5; 19], and Gregorian chant [1; 10].

Second, the analysis deliberately abstracts from historical terminology and focuses instead on the functional roles played by musical elements within their respective traditions. The objective is not to identify identical theoretical constructs or to imply a common historical origin, but rather to identify analogous organizational functions operating within different musical systems [28; 29].

Third, recurring structural similarities observed across multiple traditions are interpreted as potential indicators of functional convergence rather than historical borrowing. Convergent evolution is a well-established phenomenon in evolutionary studies, in which independently evolving systems exposed to similar environmental or functional constraints may develop analogous solutions [9; 24].

Fourth, these recurring functional similarities are generalized into a higher-level representational framework referred to in the present work as Functional Grammar of Modal Music (FGMM). The proposed abstraction is conceptually analogous to generative approaches that distinguish surface realizations from deeper organizational principles in both linguistics and music theory [8; 23].

Finally, the resulting framework is evaluated conceptually according to three criteria:

1. explanatory power with respect to established musicological theories and descriptions of modal traditions [1; 5; 10; 18; 20; 37];

2. compatibility with contemporary computational musicology and Music Information Retrieval methodologies [6; 7; 14; 21; 26; 34];

3. potential applicability to comparative musicology, algorithmic composition, and interpretable generative artificial intelligence, particularly in relation to higher-level models of musical structure and improvisation [23; 31].

The present study therefore does not attempt to validate the proposed framework empirically. Rather, its objective is to establish a theoretical foundation and formulate hypotheses suitable for future experimental verification. The authors consider this limitation acceptable at the present stage because the primary objective is the formulation of a representational language rather than the optimization of a particular algorithm.

Such verification may include computational analysis of musical corpora, pitch trajectory studies, physiological experiments involving listeners, and machine learning approaches aimed at recovering the proposed functional structures from real musical data.

Consequently, the contribution of the present work lies not in the presentation of experimental evidence, but in the identification of previously underexplored functional analogies and in the formulation of a generalized representational model capable of unifying multiple modal traditions within a common conceptual framework. The authors therefore position the present study as a theory-building contribution intended to precede and motivate future empirical investigations rather than replace them.

3. Constraints in Musical Systems

Music may be understood as a balance between freedom and constraint. A fully deterministic sequence quickly becomes predictable and monotonous, whereas a completely unconstrained sequence approaches randomness and loses coherence and recognizability [23; 31].

Consequently, every musical tradition develops mechanisms that restrict the space of permissible continuations while preserving sufficient flexibility for expression and creativity [27;28].

The simplest examples may be found in strongly pulse-based musical structures such as military marches, work songs, and various forms of dance music. In such systems, temporal regularity provides a major organizational constraint. Recurrent rhythmic patterns establish a predictable temporal framework within which substantial melodic variation may occur (7; 26).

Classical tonal music introduces an additional level of organization through harmonic constraints. Chord progressions, functional harmony, voice-leading principles, and cadential structures organize the range of plausible musical continuations and shape expectations regarding subsequent development [23].

Modal traditions introduce yet another type of organization. Rather than relying primarily on harmonic progression, modal systems organize melodic movement through privileged tones, characteristic pathways of development, and recurrent melodic formulae. These elements acquire tradition-specific functional roles and constrain stylistically plausible realizations of a mode [1; 5; 10; 18; 20; 37].

Importantly, these constraints are not merely theoretical prescriptions imposed externally upon performers. They have emerged through long historical processes of cultural refinement, performance practice, and transmission. Modal traditions such as mugham, makam, raga, and Gregorian chant have developed over extended historical periods through successive generations of performers, teachers, and theorists [13; 10; 18; 19; 27; 29; 38].

From this perspective, modal systems may be viewed as products of cultural evolution. This interpretation remains speculative and should be regarded primarily as a heuristic perspective rather than as an established musicological fact. One may hypothesize that melodic structures better aligned with the aesthetic, performative, and contemplative functions of a tradition were more likely to be retained, refined, and transmitted across generations, whereas less compatible structures gradually disappeared from established practice [27; 29].

Such an interpretation naturally leads to an important question: if multiple civilizations independently developed musical systems associated with sustained attention, meditation, prayer,

or prolonged spiritual practice, is it possible that common organizational principles emerged repeatedly despite differences in language, religion, and historical origin [1; 9; 10; 25; 30; 38]?

The present work is motivated precisely by this question.

The following classification is not intended as a rigorous musicological taxonomy but rather as a heuristic framework that motivated the present investigation. The proposed heuristic classification is summarized in Table 1.

Musical tradition	**Dominant organizational principle**
Marches and work songs	Collective synchronization
Rhythmic dance traditions	Group motor synchronization
Partner dance traditions (e.g., tango)	Interpersonal emotional coordination
Tonal-harmonic music	Management of expectation and resolution
Jazz improvisation	Sustained attention and expectation management
Modal spiritual traditions	Sustained contemplative attention

Table 1. Heuristic classification of musical traditions according to their dominant cognitive and social functions.

Admittedly, many musical traditions simultaneously fulfill several cognitive and social functions. The proposed classification merely highlights their dominant organizational tendencies.

Rather than emphasizing differences between modal traditions, we investigate the possibility that they may share a common functional architecture operating beneath their distinct theoretical vocabularies.

The following section examines this hypothesis through a comparative analysis of several major modal systems.

4. Functional Convergence of Modal Traditions

The comparative methodology outlined in the previous section suggests that independently evolved modal traditions may be analyzed in terms of the functional roles played by their constituent elements rather than their historical terminology.

When examined from this perspective, several striking similarities emerge between modal systems traditionally regarded as unrelated.

Azerbaijani mugham employs concepts such as mayə, melodic development through şöbələr, and characteristic melodic behavior associated with particular modal realizations [18].

Turkish makam theory formalizes functionally comparable principles through concepts such as durak, güçlü, seyir, and characteristic modal structures [20; 37].

Indian raga theory describes related functional roles through sa, vadi, samvadi, chalan, and pakad [5; 19; 30].

Gregorian chant, despite its distinct historical and theological background, likewise exhibits stable final tones, reciting tones, characteristic melodic formulae, and preferred patterns of modal development [1; 10].

The terminology differs substantially across traditions, and the present work does not imply common historical origin or direct transmission between these systems.

However, when historical vocabulary is replaced by functional interpretation, four recurrent organizational principles appear repeatedly.

4.1 Primary Functional Center

Every examined modal tradition contains one or more privileged reference tones that serve as points of stability and resolution.

These include mayə in mugham, durak in makam, finalis in Gregorian chant, and sa in Indian raga.

The melodic trajectory repeatedly returns to these tones, which function as anchors for perception and orientation.

4.2 Secondary Functional Center

In addition to the principal center, modal systems frequently contain secondary points of attraction that organize larger sections of melodic development.

Examples include güçlü in makam theory, reciting tones in Gregorian chant, and vadi in Indian raga.

Although their exact theoretical status varies, their functional role appears remarkably similar.

4.3 Characteristic Developmental Trajectory

Modal systems rarely permit arbitrary movement between pitches.

Instead, they define preferred directions of development and characteristic sequences of exploration and return.

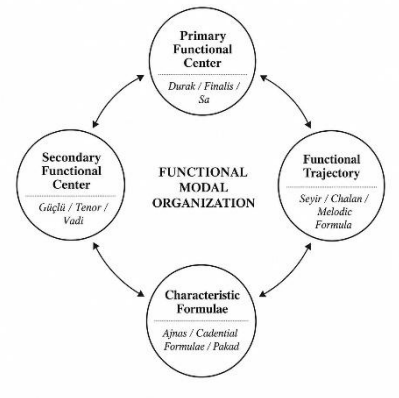
The Turkish concept of seyir represents perhaps the clearest explicit formulation of this principle, although analogous concepts can be identified in mugham development, raga chalan, and Gregorian melodic formulae.

4.4 Recurrent Melodic Patterns

Finally, all examined traditions employ recognizable melodic fragments, formulae, or motifs associated with particular modal identities.

Examples include ajnas in makam traditions, pakad in Indian raga, characteristic cadential formulae in Gregorian chant, and modal phrase structures in mugham performance practice.

These patterns contribute to stylistic recognition and continuity across performances. The four recurrent components and their functional analogies across modal traditions are summarized in Figure 1.



The repeated appearance of these four functional principles across historically independent modal traditions suggests the existence of a deeper level of musical organization that transcends local terminology and theoretical schools.

The present work refers to this common organizational layer as the Functional Grammar of Modal Music.

The following section introduces this framework formally.

5. Functional Grammar of Modal Music

The comparative analysis presented in the previous section suggests the existence of a common functional layer underlying several historically independent modal traditions.

The present work refers to this layer as the *Functional Grammar of Modal Music (FGMM).*

The term *grammar* is used here deliberately and draws on generative approaches in linguistics and music theory, in which complex structures are understood not merely as collections of elements but in terms of rules and principles governing their organization and permissible relations [8; 23].

Similarly, modal music may be viewed not simply as a sequence of pitches but as a constrained trajectory evolving within a space of permissible functional states.

Under this interpretation, individual notes cease to be the primary objects of analysis. Instead, the primary objects become the functional roles performed by musical events within the evolving structure of the melody.

FGMM proposes that modal organization can be represented through four minimal functional components:

1. Primary Functional Center;
2. Secondary Functional Center;
3. Developmental Trajectory;
4. Characteristic Melodic Patterns.

Together, these components define the functional state of the musical system at any given moment.

5.1 The Primary Functional Center

The Primary Functional Center acts as the principal point of stability and perceptual orientation. It serves as the ultimate destination of melodic resolution and provides long-term structural coherence.

Examples include *mayə* in Azerbaijani mugham [18], *durak* in Turkish makam [20;37], *finalis* in Gregorian chant [1; 10], and *sa* in Indian raga [5; 19].

The melodic trajectory repeatedly returns to this functional center, which acts as an anchor for both performer and listener.

5.2 Secondary Functional Center

The Secondary Functional Center introduces an intermediate level of organization. It creates local regions of stability and enables the construction of larger-scale musical forms.

Examples include *güçlü* in Turkish makam theory [20; 37], *reciting tones* in Gregorian chant [1; 10], and *vadi* in Indian raga [5; 19].

Although their exact theoretical interpretation differs between traditions, their functional role appears remarkably similar: they organize larger melodic sections and facilitate transitions between different regions of the modal space.

5.3 Functional Trajectory

The Developmental Trajectory defines preferred directions of melodic motion and constrains the order in which different regions of the modal space are explored.

5.4 Characteristic Patterns

Characteristic Patterns constitute local grammatical structures associated with modal identity.

Unlike individual notes, these patterns represent higher-order melodic entities recognized by performers and listeners as stylistic markers of a particular tradition.

Examples include ajnas in makam traditions, pakad in Indian raga, cadential formulae in Gregorian chant, and characteristic phrase structures in mugham performance practice.

Such patterns constrain local melodic realization while preserving substantial freedom for improvisation.

Importantly, FGMM does not prescribe the use of specific pitches, intervals, or scales. Instead, it operates at a higher level of abstraction describing relationships between functional entities rather than acoustic events themselves.

This distinction allows the same representational framework to be applied to modal traditions possessing very different pitch systems and tuning practices. The resulting representation may therefore be interpreted as a form of functional state space in which musical development corresponds to movement between regions characterized by different functional properties. Such an interpretation naturally connects traditional musicology with modern computational approaches including Music Information Retrieval, sequence modeling, and generative artificial intelligence. Most importantly, FGMM shifts the focus of analysis from the question: *"Which note follows next?"* to the more fundamental question: *"Which functional transition is permitted or preferred within the current modal context?"* This change of perspective constitutes the principal theoretical contribution of the present work.

The musical state may therefore be represented by a four-component functional state description. In the present conceptual formulation, these components should be understood as classes of state variables rather than necessarily as scalar coordinates of a Euclidean four-dimensional space.

$$S(t) = \left(C_p(t), C_s(t), T(t), P(t)\right)$$

Where:

C_p— Primary Functional Center,

C_s— Secondary Functional Center,

T— Functional Trajectory,

P— Characteristic Patterns.

Conceptually, this representation draws on the state-space perspective of analytical mechanics, in which the evolving condition of a system is described through a set of variables subject to structural constraints (Lanczos, 1986).

The modal melody may then be represented as a trajectory through the resulting functional state space:

$$\Gamma: S(t_0) \rightarrow S(t_f)$$

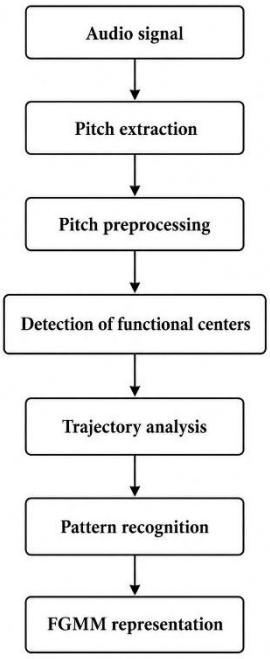
Future studies may reveal additional dimensions necessary for describing more specialized traditions or hybrid musical forms.

6. Computational Representation of FGMM

While the Functional Grammar of Modal Music is introduced as a conceptual representation of modal organization, its ultimate value depends on its applicability to computational analysis.

The central idea of the proposed framework is that functional structure is not directly observable in the audio signal itself but emerges progressively through multiple levels of abstraction.

The proposed computational pipeline is illustrated in Figure 2.



At the lowest level, the system operates on the raw audio signal obtained from recordings of modal performances.

The first processing stage consists of pitch extraction, during which the continuous audio waveform is transformed into a time-dependent pitch representation using pitch-tracking algorithms commonly employed in Music Information Retrieval research [6; 26; 34].

The resulting pitch sequence provides a symbolic description of melodic movement while preserving temporal information required for subsequent analysis.

The next stage involves the identification of functional centers. Statistical concentration of pitch occurrences, phrase endings, sustained note durations, and recurrent points of melodic return may serve as cues for detecting primary and secondary functional centers within a performance [6; 21; 34].

Once functional centers have been identified, the melodic trajectory may be analyzed in terms of movement between these regions of attraction rather than in terms of isolated pitch events.

This transformation represents one of the principal conceptual shifts introduced by FGMM. Instead of asking which note follows another note, the framework asks which functional region follows another functional region.

The subsequent stage involves the identification of characteristic melodic patterns associated with the modal identity of the performance.

These patterns may include cadential formulae in Gregorian chant [1; Hiley, 10], characteristic transitions and melodic gestures in makam traditions [20; 37], and recurrent identifying phrases in Indian raga [5; 19]. Such structures may function as markers of modal and stylistic identity within their respective traditions.

The resulting representation constitutes the Functional Grammar of Modal Music itself.

At this level, a modal performance is no longer represented as a sequence of pitches but as a trajectory through a space of functional states and transitions.

Such a representation naturally supports several classes of computational applications.

Structural analysis becomes possible through quantitative characterization of functional trajectories.

Automatic classification may exploit differences in functional organization between modal traditions.

Comparative musicology may investigate similarities and differences between independently evolved musical systems.

Finally, generative algorithms may use functional constraints to produce stylistically coherent melodic material while preserving the internal logic of a given tradition.

The authors emphasize that the present work does not claim that all elements of this pipeline are currently solved problems.

Rather, the objective is to establish a computational roadmap linking existing MIR techniques with higher-level representations traditionally employed by performers and musicologists.

In this sense, FGMM may be viewed as an intermediate representational layer situated between acoustic signal processing and musical cognition.

7. Quantitative Measures in Functional Modal Space

The Functional Grammar of Modal Music introduces a representation of modal melodies in terms of functional states and transitions rather than individual pitch events.

Such a representation naturally allows the introduction of quantitative descriptors characterizing the structure and behavior of modal trajectories.

Unlike conventional Music Information Retrieval approaches that primarily operate on pitch distributions or interval statistics, FGMM descriptors are intended to quantify higher-order organizational properties of modal development.

7.1 Functional Entropy

One possible descriptor is functional entropy. Following Shannon's information-theoretic formulation of entropy as a measure of uncertainty over a probability distribution [36], we propose applying an analogous measure to the distribution of permissible functional transitions.

At any given moment during a performance, a performer may possess multiple permissible continuations consistent with the underlying modal grammar.

Some modal contexts permit only a small number of acceptable continuations, whereas others allow considerably greater freedom.

Functional entropy may therefore be interpreted as a measure of uncertainty associated with the next permissible transition within the current modal context.

$$H(S_t) = -\sum_i p_i \log p_i$$

where p_i - is the probability of a valid transition.

Low entropy corresponds to highly constrained situations characterized by strong expectations regarding subsequent development.

High entropy corresponds to regions of increased improvisational freedom and structural variability.

7.2 Center Stability

Another important descriptor is the stability of functional centers.

Certain modal traditions maintain a strong attraction toward the primary functional center, whereas others exhibit extended exploration of secondary regions before eventual resolution.

Center stability may therefore quantify the relative strength of attraction associated with different functional regions within the modal space.

7.3 Trajectory Complexity

Modal traditions differ substantially in the complexity of their developmental trajectories.

Some traditions employ relatively direct movements between functional regions, whereas others involve long excursions, nested structures, and multiple intermediate stages.

Trajectory complexity may therefore provide a quantitative characterization of modal development independent of the specific pitch inventory employed by a given tradition.

7.4 Pattern Density

Characteristic melodic patterns constitute an important component of modal identity.

The density, diversity, and recurrence frequency of such patterns may provide additional descriptors useful for classification and comparative analysis.

7.5 Predictability

Predictability represents another potentially useful measure.

Musical systems differ in the degree to which future development may be anticipated by experienced listeners.

Excessive predictability may lead to monotony, whereas insufficient predictability approaches randomness and reduces coherence.

Modal traditions appear to operate within an intermediate region balancing novelty and expectation.

7.6 Functional Distance Between Modal Systems

The proposed representation also permits the definition of distances between modal traditions.

Rather than comparing scales or interval structures directly, FGMM allows comparison of functional organization itself.

Two modal systems may therefore differ substantially in pitch content while exhibiting surprisingly similar functional architectures.

Conversely, systems sharing similar scales may display entirely different developmental grammars.

7.7 Variational Interpretation of Modal Development

An intriguing interpretation emerges when modal development is viewed through the perspective of variational principles in analytical mechanics. In this tradition, the motion of a system may be represented in a configuration space defined by generalized coordinates and constraints, while variational methods consider admissible changes in the system's trajectory (Lanczos, 1986). To the authors' knowledge, an explicit analogy between this variational framework and modal development has not previously been explored in computational musicology.

Under such an interpretation, the performer does not choose notes individually but rather navigates through a space of permissible trajectories defined by the modal grammar. The proposed analogy is summarized in Table 2

Analytical Mechanics	**Functional Modal Music**
Generalized coordinates (q_i)	Functional state variables
State space	Functional modal space
Constraints	Modal grammar
Admissible variations	Permissible local continuations
Trajectory $q(t)$	Melodic trajectory Γ
Action functional $S[q]$	Stylistic functional $J[\Gamma]$
Principle of stationary action	Principle of stylistic optimality

Table 2. Analogy between variational mechanics and modal development.

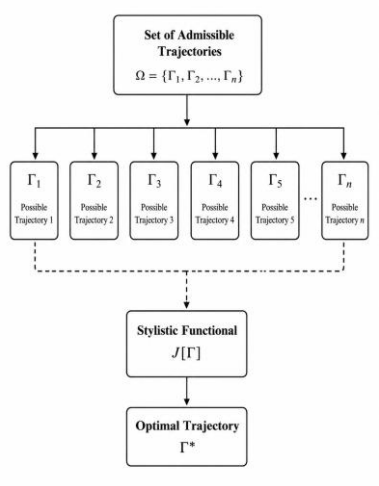
$\Gamma_1, \Gamma_2 \ldots \Gamma_n$

denote all valid continuations.

$\Gamma^* = \arg opt_\Gamma J[\Gamma]$

where $J[\Gamma]$denotes an unknown stylistic functional encoding accumulated aesthetic, cognitive, and historical preferences of the tradition.

This variational interpretation is illustrated in Figure 3.



At any given moment, a performer is typically faced with multiple stylistically acceptable continuations. The actual melodic trajectory represents only one realization among many possible alternatives.

A similar cognitive mechanism may be observed in other improvisational and interactive practices. An experienced tango dancer, for example, does not select movements independently but continuously evaluates a set of admissible continuations constrained by rhythm, partner position, balance, and stylistic conventions, ultimately selecting one trajectory among many possible alternatives.

The present work suggests that modal improvisation may operate according to a comparable principle, where the performer navigates a space of stylistically admissible melodic trajectories rather than selecting isolated notes independently.

The observed trajectory may therefore be viewed as a path minimizing or optimizing an unknown functional encoding the accumulated aesthetic, stylistic, and historical preferences of the musical tradition.

Importantly, the present work does not attempt to specify the mathematical form of this functional. Indeed, one may speculate that such a functional could be too complex to derive analytically and might instead be recoverable only through machine learning from sufficiently large musical corpora.

Instead, it is hypothesized that modern machine learning methods may be capable of recovering such structures directly from sufficiently large corpora of performances.

In this sense, FGMM may provide the state space within which the hidden objective function governing modal development operates.

7.8 Implications for Generative Artificial Intelligence

The variational interpretation naturally suggests a new approach to algorithmic composition and generative artificial intelligence.

Instead of generating melodies through local statistical continuation of pitch sequences, future systems may generate trajectories satisfying the functional constraints of a given modal tradition.

Such an approach would allow artificial systems not merely to imitate existing melodies but potentially to participate in the same functional grammar that guided generations of performers and composers.

The authors emphasize that this possibility should not be interpreted as an attempt to replace human creativity.

Rather, the objective is to understand, preserve, and formalize the accumulated musical knowledge embedded within centuries of cultural evolution.

8. Discussion

The proposed Functional Grammar of Modal Music should not be interpreted as a replacement for existing musicological theories or as an attempt to reduce rich musical traditions to a small set of computational descriptors.

Rather, FGMM is intended as an additional representational layer situated between low-level acoustic features and high-level musical cognition.

The central idea of the framework is that modal traditions may share common organizational principles despite substantial differences in terminology, tuning systems, historical development, and cultural context.

From this perspective, the proposed framework is conceptually closer to comparative linguistics or comparative anatomy than to conventional music classification approaches.

Languages may differ dramatically in vocabulary and grammar while still exhibiting common structural principles. Similarly, biological organisms may evolve independently while converging toward analogous functional solutions.

The present work suggests that modal musical traditions may exhibit a comparable form of functional convergence.

An important consequence of this interpretation concerns improvisation.

Experienced performers rarely select individual notes in isolation. Instead, musical decisions appear to be guided by internalized representations of permissible trajectories, characteristic gestures, and functional regions within the modal space.

FGMM may therefore be interpreted not only as a computational representation but also as a possible approximation of cognitive mechanisms employed during modal improvisation. Whether human performers explicitly represent such structures consciously remains an open question.

Another implication concerns the relationship between musicology and artificial intelligence.

Recent generative models have made substantial progress in capturing long-range musical structure and providing high-level control over symbolic music generation [17; 40]. However, these advances do not in themselves provide an explicit representation of tradition-specific functional roles such as primary and secondary centers, characteristic developmental trajectories, and recurrent modal patterns. FGMM is proposed as a complementary functional layer that may improve the interpretability and controllability of modal music generation.

The framework may also contribute to preservation of intangible cultural heritage.

Many modal traditions are transmitted primarily through oral pedagogy and master-apprentice relationships.

Formal representation of their underlying organizational principles may facilitate documentation, comparative analysis, education, and digital preservation without replacing traditional performance practice.

The authors acknowledge several limitations of the present study.

First, the proposed framework has not yet been validated using large musical corpora.

Second, the proposed set of four functional components should not be interpreted as necessarily complete or universally minimal. The present formulation should therefore be regarded as a first approximation rather than a definitive taxonomy of modal organization.

Additional functional dimensions may emerge as the framework is applied to a broader range of modal traditions.

Third, the present work intentionally focuses on melodic organization and therefore does not address rhythmic structure, timbral characteristics, textual content, or performance practice, all of which may play important roles in modal identity.

These limitations should be regarded not as weaknesses of the approach but rather as directions for future investigation.

The authors therefore view FGMM as a theory-building contribution intended to stimulate empirical research rather than as a completed theory of modal music.

9. Conclusion

The present work introduced the concept of Functional Grammar of Modal Music (FGMM), a generalized framework for representing modal musical traditions in terms of functional states, constraints, and transitions.

By shifting attention from individual pitch events toward higher-level organizational principles, the proposed approach seeks to bridge the gap between traditional musicology, computational analysis, and generative artificial intelligence.

Comparative analysis suggests that several historically independent modal traditions, including mugham, makam, raga, and Gregorian chant, may share a common functional architecture despite substantial differences in terminology and historical development.

The framework identifies four recurrent organizational components: a primary functional center, a secondary functional center, a developmental trajectory, and a system of characteristic melodic patterns.

Together, these components form the basis of a generalized representational language for modal organization.

The study further proposes that modal development may be interpreted as movement through a space of permissible trajectories governed by functional constraints.

This perspective opens new possibilities for structural analysis, automatic classification, cross-cultural comparison, and algorithmic generation of stylistically coherent modal material.

The proposed framework remains theoretical and requires empirical validation through computational analysis of musical corpora and machine learning approaches capable of recovering functional structures directly from musical data.

Nevertheless, the authors believe that the identification of previously underexplored functional analogies between independent modal traditions represents a promising direction for future research at the intersection of musicology, cognitive science, and artificial intelligence.

Figure Captions and Alt Text

Figure 1 Caption: Functional convergence of modal traditions and the four fundamental components of modal organization.

Figure 1 Alt Text: Circular scheme linking primary and secondary functional centers, functional trajectory, and characteristic formulae as four interacting components of modal organization across different traditions.

Figure 2 Caption: Computational pipeline for transforming audio recordings into Functional Grammar of Modal Music representations.

Figure 2 Alt Text: Vertical pipeline progressing from audio signal through pitch extraction, pitch preprocessing, functional-center detection, trajectory analysis, and pattern recognition to an FGMM representation.

Figure 3 Caption: Variational interpretation of modal development. Among multiple stylistically admissible trajectories, the performed melody corresponds to an optimal trajectory with respect to an implicit stylistic functional.
Figure 3 Alt Text: Multiple admissible melodic trajectories are evaluated by an implicit stylistic functional, which selects an optimal trajectory from the set of stylistically permissible alternatives.

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MODAL MUSİQİNİN FUNKSİONAL QARAMMATİKASI: MODAL TƏŞKİLATLANMANIN VAHİD NƏZƏRİYYƏSİNƏ DOĞRU

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Xülasə
Təqdim olunan araşdırmada səs ucalığı ardıcıllığının akustik və ya statistik xüsusiyyətlərinə görə əsaslandırılan modal musiqinin funksional təşkilatlanmasına dair nəzəri konsept təklif olunur. Əhəmiyyətli tarixi və mədəni fərqlərə baxmayaraq, Azərbaycan muğamı, türk məqamı, hind raqası və qriqorian nəğməsi dayaq tonları, ikinci dərəcəli dayaq mərkəzləri, xarakterik trayektoriyalar və təkrarlanan strukturlar ətrafında melodik inkişafın təşkilində funksional analogiyalar nümayiş etdirir. Musiqi məlumatlarının tədqiqi sahəsində mövcud yanaşmaların əksəriyyəti akustik xüsusiyyətlər, səs yüksəkliyinin paylanması və ya statistik qanunauyğunluqlarla işləyir. Məqalənin məqsədi modal musiqinin funksional qrammatikasını (FGMM – Functional Grammar of Modal Music) təqdim etməkdir ki, məqalə müəllifləri dörd minimal komponenti – ilkin funksional mərkəz, ikinci dərəcəli funksional mərkəz, inkişaf trayektoriyası və təkrarlanan melodik patternləri fərqləndirərək, modal musiqinin funksional qrammatikasını (FGMM) daxil edirlər. Bu metod hər hansı bir konkret musiqi ənənəsindən asılı olmayaraq işlənib hazırlanmışdır və struktur analizi, avtomatik təsnifatı, mədəniyyətlərarası müqayisəni, elecə də interpretasiya oluna bilən alqoritmik generasiyanı dəstəkləmək daşıyır. Təklif olunan metod tam başa çatdırılmış hesablama sistemi kimi deyil, empirik yoxlama üçün konseptual baza kimi nəzərdən keçirilməlidir. O, tarixi baxımdan bir-birindən asılı olmayan modal ənənələr arasında az öyrənilmiş funksional analogiyaların üzə çıxarılmasına yönəlmişdir.
Açar sözlər: modal musiqi, funksional qrammatika, musiqi məlumatlarının axtarışı, hesablama musiqişünaslığı, melodik trayektoriyalar, generativ süni intellekt.

ФУНКЦИОНАЛЬНАЯ ГРАММАТИКА МОДАЛЬНОЙ МУЗЫКИ: НА ПУТИ К ЕДИНОЙ ТЕОРИИ МОДАЛЬНОЙ ОРГАНИЗАЦИИ

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Резюме
В настоящем исследовании предлагается теоретическая основа для функциональной организации модальной музыки на основе акустических или статистических свойств звуковысотной последовательностей. Несмотря на существенные исторические и культурные различия, азербайджанский мугам, турецкий макам, индийская рага и григорианское пение демонстрируют функциональные аналогии в организации мелодического развития вокруг опорных тонов, вторичных опорных центров, характерных траекторий и повторяющихся мелодических структур. Существующие подходы к

исследованию музыкальной информации (MIR) в большинстве своем работают с акустическими характеристиками, распределением звуковысотности или статистическими закономерностями, на этом основании оставаться ограничены в охвате функциональной логики более высокого уровня. Для решения этой проблемы авторы статьи вводят функциональную грамматику модальной музыки (FGMM), выделяя четыре минимальных компонента: первичный функциональный центр, вторичный функциональный центр, траекторию развития и повторяющиеся мелодические паттерны. Данный метод разработан независимо от какой-либо отдельной музыкальной традиции и предназначен для поддержки структурного анализа, автоматической классификации, межкультурного сравнения и интерпретируемой алгоритмической генерации. Предлагаемый метод следует рассматривать как концептуальную и математическую основу для эмпирической проверки, а не завершенную вычислительную систему. Он направлен на выявление малоизученных функциональных аналогий между исторически независимыми модальными традициями.

Ключевые слова: модальная музыка, функциональная грамматика, поиск музыкальной информации, вычислительная музыковедение, мелодические траектории, генеративный искусственный интеллект.

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